# NINETY-SIXTH INTERNATIONAL ATLANTIC ECONOMIC CONFERENCE

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*"On Digital Currencies"*

*5-8 OCTOBER 2023*

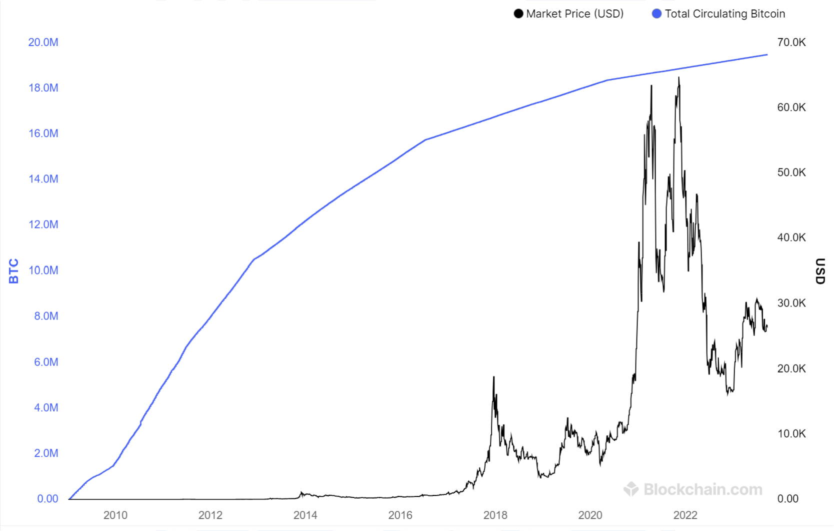
# This talk:

1. Theory of Cryptocurrency Pricing: **Schilling-Uhlig, “Some Simple Bitcoin Economics”, JME 2019.**
2. Central bank digital currencies: **Schilling - Fernández-Villaverde - Uhlig, “CBDC: when Price and Bank Stability Collide”, WP.**

# The Origin

Satoshi Nakamoto (2008), “Bitcoin: A Peer-to-Peer Electronic Cash System.”

# Bitcoin Quantity and Price, 2009-01-09 to 2023-09-26




Source: https://www.blockchain.com/charts/total-bitcoins

# Schilling - Uhlig, “Some Simple Bitcoin Economics”

**Key Questions:**

1. What determines the Bitcoin price? $P > \mathrm{NPV}(\mathrm{Dividends}) = 0.$?
2. Can Bitcoin serve as medium of exchange, despite price volatility?
3. What are monetary policy implications?

**Key Insights:**

1. A **novel model** of an endowment economy with two intrinsically worthless currencies (Dollar, Bitcoin) as medium of exchange.
2. “Fundamental pricing equation”. Special case: **Bitcoin price is martingale.**
   - Kareken and Wallace (1981)
   - Manuelli and Peck (1990)
3. **“No speculation” theorem.**
4. **Volatility does not invalidate medium-of-exchange function.**
5. **Monetary policy implications:**
   - **Bitcoin block rewards are** not a tax on Bitcoin holders: they are **financed with a Dollar tax.**

# The Schilling - Uhlig (2019) model

- Time: discrete, infinite $t = 0, 1, 2, \ldots$.
- Randomness: $\theta_t$, at beginning of period. History: $\theta^t$.
- One perishable consumption good per period.
- Two monies: Bitcoins $B_t$ and Dollars $D_t$ (aggregates).
- A central bank steers quantity of Dollars per lump sum transfers: $D_{t+1} = D_t + \tau_{t+1},\ \tau_{t+1} \in \mathbb{R}$. Goal: exogenous price path $P_t$.
- Bitcoin quantity: deterministic $B_{t+1} = B_t + A_t,\ A_t \geq 0$ (endowment or "mining")
- $P_t$ price of consumption good in \$: exogenous.
- $Q_t = Q(\theta^t)$ price of Bitcoins in \$: **endogenous**.
- Two types of agents: "red" and "green". "Red" agents consume in odd periods and have endowments in even periods. "Green" agents other way around.
- Goods are traded for monies. Agents do not need to spend all money ("hodlers") or accept all money. **But:** "No speculation" result: they will!

# The Fundamental Pricing Equation

Compare to Kareken-Wallace (1981), Manuelli-Peck (1990)

### Proposition 1

Assume agents use both Dollars **and** Bitcoins to buy goods at $t$ and $t+1$. Then

$$\mathbb{E}_t \left[ u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[ \left( u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right) \frac{Q_{t+1}}{Q_t} \right] \quad (1)$$

If production (consumption) is constant at $t+1$ or if agents are risk-neutral, and if further $Q_{t+1}$ and $1/\pi_{t+1}$ are conditionally uncorrelated, then **the Bitcoin price $Q_t$ in Dollar is a martingale,**

$$Q_t = E_t[Q_{t+1}]$$

# Bitcoin block rewards are financed by Dollar taxes

Consider two economies, which differ in the growth paths for the Bitcoin quantity.

- The central bank seeks to achieve the same path for prices.
- Quantity theory:

$$P_t y_t = D_t + Q_t B_t$$

- More Bitcoins $B_t$ means less $D_t$, keeping everything else the same.
- Same equilibrium can obtain, otherwise.

# S-FV-U, “CBDC: When Price & Bank Stability Collide”

**In our model:** Only HH, CB, projects. CB is financial intermediary.

**Key Mechanism**

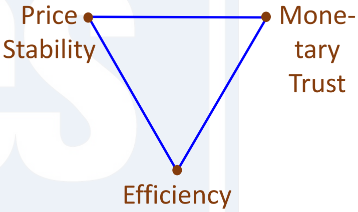
- Nominal Diamond-Dybvig (1983) model for a CB and its CBDC.
- Central bank can always deliver on its nominal obligations.
- But: CB runs can happen: “spending run” on available goods.

**Three competing objectives:**

1. Traditional CB objective: commitment to **Price Stability**
2. Social optimum, optimal risk sharing: **Efficiency**
3. Absence of runs, financial stability: **Monetary Trust**

**Key Result:** **CBDC Trilemma**

Of the three objectives, the central bank can only achieve two.



# The model: the real portion is Diamond-Dybvid, 1983

- time $t = 0, 1, 2$.
- Continuum $[0, 1]$ of agents:
  - $t = 0$: symmetric, endowed with one unit of a real good
  - $t = 1$: types reveal: "impatient" $\lambda$, "patient" $1 - \lambda$.
    Impatient agents: have to consume in $t = 1$.
  - $u(\cdot)$ strictly increasing, concave, RRA greater than one,
    $-x \cdot u''(x)/u'(x) > 1$.
- Real Technology:
  - long term: $1 \to 1 \to R$
  - storage $t = 1 \to t = 2$, available to all: $1 \to 1$
- **Optimal solution**:

$$\max \lambda u(x_1) + (1 - \lambda) u(x_2) \text{ s.t. } \lambda x_1 + (1 - \lambda)\frac{x_2}{R} = 1$$

  Unique solution, where $u'(x_1^*) = R u'(x_2^*)$
- With that: $x_1^* > 1$. (Diamond and Dybvig, 1983)

# The model: the nominal portion introduces CBDC.

$t = 0$:
- Agents sell goods to CB for M CBDC units in $t = 1$.
- CB: invests all received real goods in projects.

$t = 1$:
- Agents learn type. Impatient agents spend $M$. Patient agents may. Total fraction: $\lambda \leq n \leq 1$.
- CB **observes agg. spending fraction $n$.**
- CB liquidates fraction $y = y(n) \in [0, 1]$ of projects.
- CB sells goods $y$. Market clearing price $P_1$.

$t = 2$:
- Remaining agents spend $(1 + i(n))M$.
- CB sells remaining project payoffs $R(1 - y)$
- Market clearing price $P_2$.

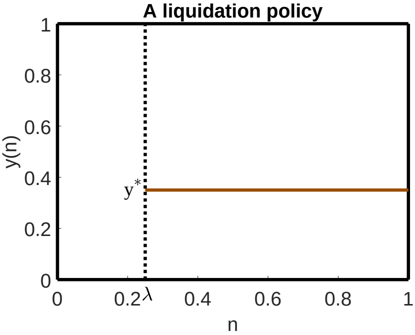
**Definition 1**

A **central bank policy** is a triple $(M, y(\cdot), i(\cdot))$, where $y : [0, 1] \to [0, 1]$ is the central bank's liquidation policy for every observed fraction $n$ of spending agents, and $i : [0, 1] \to [-1, \infty)$ is the nominal interest rate policy.

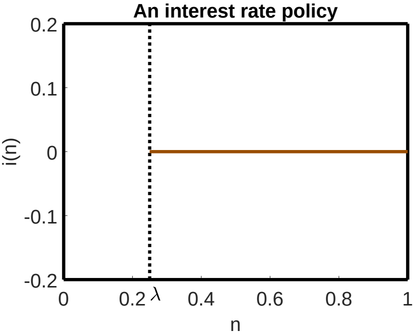
# A (boring) example for a central bank policy

A **central bank policy** is a triple $(M, y(\cdot), i(\cdot))$:

$y(n)$




$i(n)$




Set $M$ so that $P_1 = 1$ clears the market, if $n = \lambda$ agents spend in $t = 1$.

# Market Clearing

$$\begin{aligned} nM &= P_1 y(n) \\ (1-n)(1+i(n))M &= P_2 R(1-y(n)), \end{aligned}$$

$\Rightarrow n, y(n), i(n)$ pin down the price levels $P_1, P_2$.

$$P_1(n) = \frac{nM}{y(n)} \quad \text{and} \quad P_2(n) = \frac{(1-n)(1+i(n))M}{R(1-y(n))}$$

Note: $P_2(n)$ can be "anything" per $i(n)$, but $i(n)$ does not affect $P_1(n)$.
**Real allocation: only depends on $n$ via $y(n)$:**

$$x_1(n) = \frac{M}{P_1} = \frac{y(n)}{n} \quad \text{and} \quad x_2(n) = \frac{(1+i(n))M}{P_2} = \frac{1-y(n)}{1-n}R$$

**Given $n$, patient agents run iff $x_1(n) \geq x_2(n)$.**

# Objective 2: Optimal Risk Sharing

The social optimum $(x_1^*, x_2^*)$ is an equilibrium, if $y(\lambda) = y^* = \lambda x_1^*$.

# Objective 3: Absence of Runs

**A Run on the Central Bank is a Spending Run:**

**Definition 2**

A **run** occurs if $n > \lambda$: patient agents also spend.

**CBDC looses its 'store of value' function.**

- Patient agents purchase goods instantaneously even though they do not need to consume
- Enable future consumption by storing toilet paper and other goods at home rather than storing value in form of CBDC
- Trust in monetary system and CBDC evaporates.
- Monetary instability.
- Compare to:
  - temporary pandemic stockouts.
  - hyperinflations.
  - currency crises.

# No run

t=0

t=1


t=2

# Run

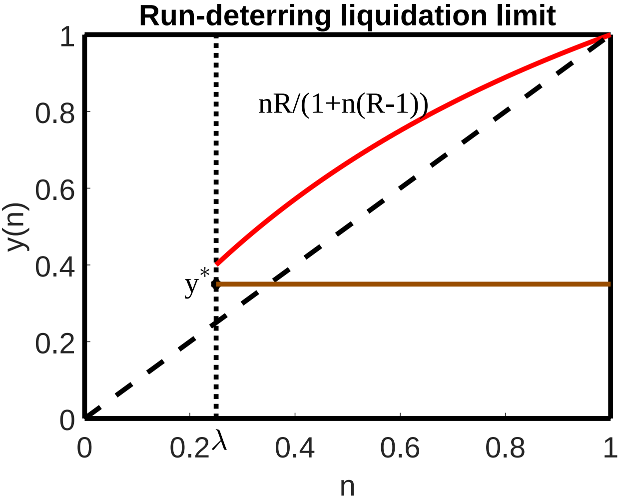
t=0

t=1


t=2

# Run-Proof Policies

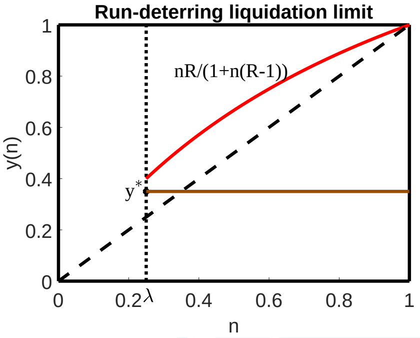
The policy is "run-proof", if $n \neq \lambda$ is "off equilibrium", i.e. if $x_1(n) < x_2(n)$ for all $n$, i.e. $y(n) < \bar{y}(n) = nR/(1 + n(R - 1))$.

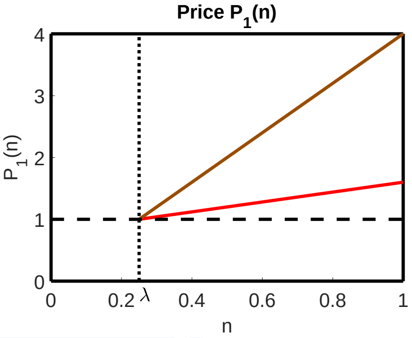



Example: the policy $y(n) \equiv y^*$ is run-proof.

# Run-Proof Policies and Price Implications

E.g. for $y(n) \equiv y^*$, we have $P_1(n) = n\frac{M}{y^*}$




- These two policies violate the price stability objective for $P_1(n)$.
- The problem only arises "off equilibrium."
- Commitment-issue / credibility / sub-game perfection: should $n \neq \lambda$ arise, a price-stability oriented Central Bank may not stick to the "threat" of letting the price $P_1$ move far from the target.
- Remark: objective for $P_2(n)$ can always be achieved via $i(n)$.

# Objective 1: Price Stability

**Definition 3**

1. A central bank policy is **fully price stable**, if it achieves $P_1(n) \equiv \overline{P}$ for all $n$.
2. A central bank policy is **partially price stable**, if it achieves **either** $P_1(n) = \overline{P}$ **or** there is full liquidation, $y(n) = 1$, for all $n$.

( In the paper: extend to period 2, pick the right interest rate policy. )
Recall Market Clearing:

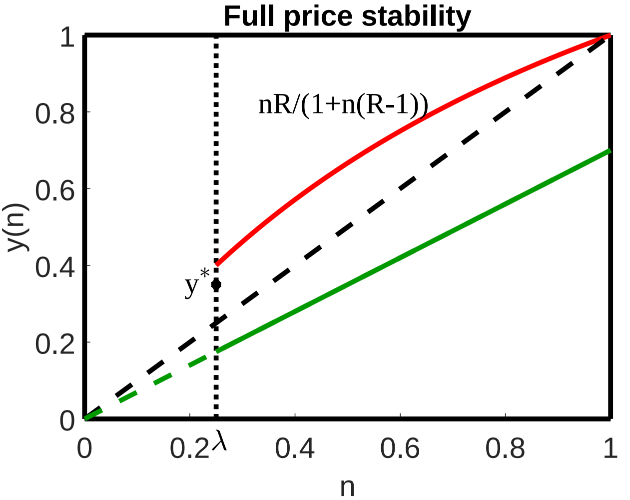
$$P_1(n) = \frac{nM}{y(n)}$$

Therefore,

**fully price stable:** $$y(n) = \frac{nM}{\bar{P}}$$

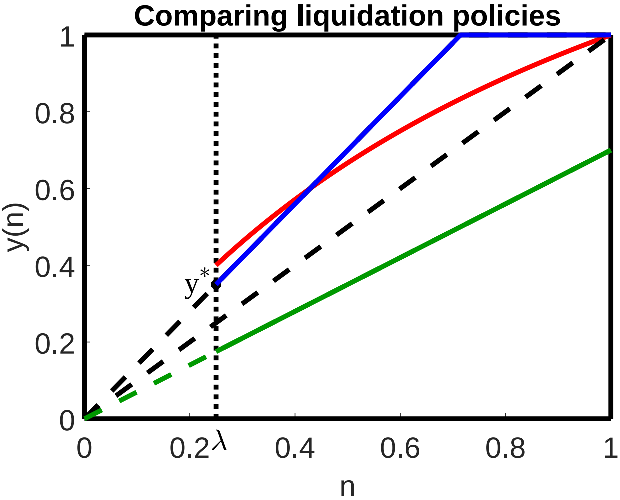
**partially price stable:** $$y(n) = \min\{\frac{nM}{\bar{P}}\}$$

# Full price stability




- Prices are fully stable and runs are avoided on green line.
- **But:** no longer efficient at $n = \lambda$!
- At best: green line = 45 degree line.

# Partial price stability




- Prices partially stable and efficiency on blue line.
- **But:** no longer run-proof. Runs may happen!

# Conclusions

Two papers

1. Theory of Cryptocurrency Pricing: **Schilling-Uhlig, "Some Simple Bitcoin Economics", JME 2019.**
2. Central bank digital currencies: **Schilling - Fernández-Villaverde - Uhlig, "CBDC: when Price and Bank Stability Collide".**

... but a lot more can and should be done! Wide-open canvas.

**THANKS!**

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