## Dynamic Models of Photovoltaic System Installations and Upgrades

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I wish to thank my thesis adviser, Paul Scott, for his advice and patience throughout this project. I also gratefully acknowledge support from the Dean's Undergraduate Research Fund (DURF) at NYU.

## **Motivation**

Most literature treats solar energy adoption as a terminal action

Real world: option to upgrade leads to "stopping problem"

How does that change estimates of installation rates, effects of incentives, etc.?

Probability of agents upgrading their existing photovoltaic system?



### Literature

De Groote and Verboven (2019) account for the intertemporal nature of solar energy adoption in their DDC model. Find that consumers excessively discount the future, so upfront incentives are better.

Langer & Lemoine's (2022) DDC model studies optimal dynamic subsidy paths in CA. Find the most effective subsidy should increase over time as such a path effectively avoids over subsidizing agents.



## Data

Lawrence Berkeley Lab's Tracking the Sun Report includes more than 2 million PV systems.

Focus:

County level solar panel installation data from California, Connecticut, Massachusetts & Minnesota

Non third party owned residential P.V. systems

Installed between 2007 and 2021



Variable	Min	Max	Mean	Median	Std
New Installations	115.00	116990.00	73844.98	86553.00	36284.72
Expansions	0.00	7839.00	2647.38	2025.00	2534.03
Price per kW	0.00	545978.18	4862.47	4389.78	2532.92
Rebate per kW	0.00	12034.91	168.53	0.00	554.80
System Size	0.04	4043.66	7.32	6.46	11.11
Module Efficiency	0.07	0.23	0.19	0.19	0.02

#### Table 1: Summary Statistics of Key Variables







### **Investment in Upgrades**





# Incident Energy & Adjusting size

$$IE_c = \frac{\text{Mean size } (\text{kW})_{c,y}}{\eta_{c,y}}$$

Then, for every year t,

Effective Size<sub>c,t</sub> =  $\eta_{c,t} \cdot IE_c$ 

y= first observed year

 $\eta$ = system efficiency (in %)



### **Net Present Value Variable**

 $NPV = (\frac{1}{1-\beta}$  Electricity savings) – Installation cost net of incentives

Accounts for:

- 1. Improvement in system efficiency (relative to first year)
- 2. Installation costs
- 3. Subsidy received (upfront)
- 4. Tax benefits (ITC)
- 5. Electricity savings from net metering



### Relationship between NPV & P(install)





### Relationship between NPV & Pr(upgrade)





# **Dynamic Model #1- Terminal action**

Profit function 
$$\pi_t(d;\theta) = \begin{cases} a + b(NPV)_t & \text{if } d_t = 1 \text{ i.e. agent decides to install} \\ 0 & \text{if } d_t = 0 \text{ i.e. agent decides to wait} \end{cases}$$

Per period utility 
$$u_t(d,\epsilon;\theta) = \begin{cases} a+b(NPV_t)+\epsilon(d_t) & \text{if } d_t=1\\ 0+\epsilon(d_t) & \text{if } d_t=0 \end{cases}$$

Choice specific value function  $\tilde{v}_t(d;\theta) = \begin{cases} a + b(NPV_t) + \frac{\gamma}{1-\beta}, & \text{if } d_t = 1 \\ \beta EV_{t+1}(d,\epsilon;\theta) & \text{if } d_t = 0 \end{cases}$ 



#### Optimal Stopping Problem

$$V_t(\epsilon;\theta) = \max_{\{d_t,d_{t+1},\ldots\}} \mathbb{E}\left[\sum_{j=t}^{\infty} \beta^{j-t} u_j(d;\theta_j) \Big| NPV_t\right]$$

Bellman Equation reformulation

$$V_t(\epsilon; \theta) = \max_{d_t} \left[ u_t(d, \epsilon; \theta) + \beta E V_t(d, \epsilon; \theta) \right]$$

$$EV_t(d,\epsilon;\theta) = ln(\sum_d exp(\tilde{v_t}(d;\theta))) + \gamma$$

Last period T

$$ilde{v_T}(d; heta) = \left\{egin{array}{cc} a+b(NPV_T)+rac{\gamma}{1-eta}, & ext{if } d_T=1 \ \ eta EV_T(d,\epsilon; heta), & ext{if } d_T=0 \end{array}
ight.$$



#### Solve for period T then iterate backwards

Model predicts:

$$\tilde{Pr}(install)_t = \tilde{Pr}(d=1;\theta)_t$$

$$=rac{exp( ilde{v}_t(d=1; heta))}{exp( ilde{v}_t(d=1; heta))+exp( ilde{v}_t(d=0; heta))}$$



## **Estimation**

Static estimates (2021 data) as initial guess:

a= -15.4, b= 3.8e-4

Maximize log likelihood of observing the predicted probabilities for each county, then sum over counties

$$l(N, n, \tilde{Pr}, t) = \prod_{t} binom(N_t, \tilde{Pr}_t) = \prod_{t} {N_t \choose n_t} \tilde{Pr}(install)_t^{n_t} (1 - \tilde{Pr}(install)_t)^{N_t - n_t}$$

N= owner occupied households , n= new installations,  $\tilde{Pr}$  = Pr(new installation)



# Dynamic Model #2- Upgrade Option

y= first year with observed expansions  $\eta_{y-1} = 0$  and  $NPV_{y-1} = 0$ Agents start with (y-1) vintage or, equivalently, no solar panels Upgrading from a (y-1) vintage = installing P.V. system for the first time

In year t with vintage i:

- $d_t = \begin{cases} 1 & \text{if agent decides to upgrade to vintage } t \\ 0 & \text{if agent decides to keep existing vintage } i \end{cases}$



Profit function 
$$\pi_{t,i}(d;\theta) = \begin{cases} a + b(NPV'_t - NPV'_i) - IC_i & \text{if } d_t = 1 \\ m\mathbb{1}[i \neq y - 1] & \text{if } d_t = 0 \end{cases}$$

$$\begin{array}{ll} \text{Per period utility} \quad u_{t,i}(d,\epsilon;\theta) = \begin{cases} a + b(NPV'_t - NPV'_i) - IC_i + \epsilon_t(d=1) & \text{if } d_t = 1 \\ \\ m\mathbbm{1}[i \neq 2006] + \epsilon_t(d=0) & \text{if } d_t = 0 \end{cases}$$

$$\begin{array}{ll} \text{Choice specific} \\ \text{value function} \end{array} \quad \tilde{v}_{t,i}(d;\theta) = \left\{ \begin{array}{ll} a + b(NPV'_t - NPV'_i - IC_i) + \beta EV_{\theta,t+1,t} & \text{if } d_t = 1 \\ \\ m\mathbbm{1}[i \neq 2006] + \beta EV_{\theta,t+1,i} & \text{if } d_t = 0 \end{array} \right.$$



#### Optimal Stopping Problem

$$V_{t,i}(\epsilon_t;\theta) = \max_{\{d_t,d_{t+1},\ldots\}} \mathbb{E}\left[\sum_{j=t}^{\infty} \beta^{j-t} u_{j,i}(d;\theta) \Big| NPV_t, NPV_i, IC_i\right]$$

Bellman Equation reformulation

$$V_{t,i}(\epsilon;\theta) = \max_{d_t} \left[ u_{t,i}(d,\epsilon;\theta) + \beta E V_{t,i}(d,\epsilon;\theta) \right]$$

$$EV_{t,i}(d,\epsilon;\theta) = ln(\sum_{d_t} exp(\tilde{v}_{t,i}(d;\theta))) + \gamma$$

Last period T+1

$$\tilde{v}_{T+1,i}(d;\theta) = \begin{cases} a + b(NPV'_T - NPV'_i - IC_i) + \beta EV_{T+1,T}(d,\epsilon;\theta) & \text{if } d_T = 1 \\ \\ m\mathbb{1}[i \neq 2006] + \beta EV_{T+1,i}(d,\epsilon;\theta) & \text{if } d_T = 0 \end{cases}$$



Model predicts:

$$\tilde{Pr}(\text{upgrade in } t | \text{vintage } i) = \tilde{P}_{up,t}(i) = \tilde{Pr}(d = 1 | X_{t,i}; \theta)$$

$$= \frac{exp(\tilde{v}_{t,i}(d=1;\theta))}{exp(\tilde{v}_{t,i}(d=1;\theta)) + exp(\tilde{v}_{t,i}(d=0;\theta))}$$

$$\tilde{Pr}(upgrade)_t = \sum_{i=y}^t \tilde{Pr}(upgrade \text{ in } t \cap \text{ have vintage } i)$$
  
$$= \sum_{i=y}^t \tilde{P}_{up,t}(i) \cdot \mu_t(i)$$

 $\mu_t(i)$  = share of agents in year t that have vintage i



## **Estimation**

Static estimates (2021 data) as initial guess:

a= -8.3, b= 1.28e-4

Maximize log likelihood of observing the predicted probabilities (initial installations - Pr and upgrades- q) for each county, then sum over counties

$$L(N, N', n, u, \tilde{Pr}, q, t) = \prod_{t=2007}^{2021} \binom{N_t}{n_t} \tilde{Pr_t}^n (1 - \tilde{Pr_t})^{N-n} \prod_{t=2008}^{2021} \binom{N'_t}{u_t} q_t^{u_t} (1 - q_t)^{N'_t - u_t}$$

N= owner occupied households , n= new installations,  $\tilde{Pr}$  = Pr(new installation)

N'= installed base of P.V systems, u= upgrades, q= Pr(upgrade)



### **Parameter Estimates**

**Terminal Action Model** 

Model with Upgrade Option

a= disutility= 12.666

b= NPV effect= 3.361e-07

a/b = implied installation **benefit** (in \$)

= 37,667,360

a= -1.487

b= 1.538e-06

m= -0.6986\$

Implied installation cost = a/b = -\$966,840



### **Policy implications**

#### Inelasticity to NPV:

a) no difference due to modeling choices in policies that shift NPV returnsb) counterintuitive but consistent- more work required!

#### Installation cost vs benefit:

- a) parameters are biased from ignoring the possibility of upgrades
  - policies that compensate for disutility
  - measures of switching costs
- b) large magnitudes consistent



# **Thank You!**

### **Questions/comments/feedback**

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